The acronym "NOTA" stands for "None Of The Above". You may find this information useful: The standard form of a conic centered at (h, k) is given by: $A(x-h)^2 + B(x-h)(y-k) + C(y-k)^2 = 1$, and if the conic is an ellipse, its area is given by $\frac{2\pi i}{\sqrt{B^2-4AC}}$ where $i = \sqrt{-1}$.

1. The conic $x^2 + 2x + y^2 - 6y = 0$ has its center located at (h, k) and has an area of $A\pi$ units. What is $Ah + k$?

A. $-10\pi + 3$ B. -7 C. 7 D. $10\pi - 3$ E. NOTA

2. What is the distance between the lines $x + 2y = 5$ and $x + 2y = -5$?

A. $\frac{10}{\sqrt{3}}$ B. $\frac{10}{\sqrt{5}}$ C. $\frac{10}{\sqrt{7}}$ D. 10 E. NOTA

3. What is the area enclosed by the polar cure given by $r = 2018$?

A. 0 B. $1006^2\pi$ C. 2018^2 D. $2018^2\pi$ E. NOTA

4. The directrices of the ellipse $3x^2 - 6x + y^2 - 16y + 58 = 0$ are $y = p$ and $y = q$, where $p > q$. What is $p - q$?

A. $16 - \frac{18}{\sqrt{6}}$ B. 16 C. $-\frac{18}{\sqrt{6}}$ D. $\frac{18}{\sqrt{6}}$ E. NOTA

5. Give the eccentricity of 2018! $(x - 2018e)^2 + 2018^{\pi}(y - 2018)^2 = 2018^{\pi}y^2 + 2018!!$.

A. 0 B. π C. e D. 1 E. NOTA

6. The points z in the complex plane satisfying the equation $\big||z-1| - |z-i|\big| = 2$ coincide with which of the following types of curves?

A. Hyperbola B. Catenary C. Witch of Agnesi D. Ellipse E. NOTA

7. Give the set of **all** vectors that are perpendicular to $2\hat{\imath} + 3\hat{\jmath} - 7\hat{k}$. Let $t, v \in \mathbb{R}$.

A. $\langle 2t, t, t\rangle$ B. $\langle 3v, 7t - 2v, 3t\rangle$ C. $\langle -2t, -3v, 7\rangle$ D. $\langle t, v, 2t + 3v\rangle$ E. NOTA

8. Which of the following statements are true regarding conic sections?

   I. A degenerate parabola is a pair of parallel lines, two coinciding lines, or ∅.

   II. A degenerate ellipse is a point, or ∅.

   III. A degenerate hyperbola is a pair of intersecting lines.

   IV. A non-degenerate hyperbola cannot be a function.

A. I, II, III, IV B. II, IV C. I, II, III D. I, II E. NOTA

9. Let $n \geq 2018$ and be an even integer. There are n points evenly spaced out on a circle. Three distinct points are chosen at random. What is probability that the points form a right triangle?

A. $\frac{1}{n}$ B. $\frac{3}{n}$ C. $\frac{1}{n-1}$ D. $\frac{3}{n-1}$ E. NOTA

10. There are 2018 points evenly spaced out on a circle. Three distinct points are chosen at random. The probability that the points form an acute triangle is $\frac{a}{b}$ where a and b are relatively prime integers. What is $b - a$?

A. 5041 B. 4337 C. 3329 D. 3027 E. NOTA

For questions 11-13 refer to the following: A conic section contains the point $(1,0)$ and has a focus at $(1,1)$ with its corresponding directrix with equation $3x - 4y = 1$.

11. What is the eccentricity of the conic section?

A. $\frac{2}{5}$ B. $\frac{1}{5}$ C. $\frac{5}{2}$ D. $\frac{1}{2}$ E. NOTA

12. The equation of the conic section is given as $Ax^2 + Bxy + Cy^2 + Dx + Ey - 7 = 0$. What is $A + B + C + D + E$?

A. 13 B. 11 C. 7 D. 4 E. NOTA

13. The conic's directrices are rotated by $\arctan(\frac{4}{3})$ about the origin and are given by $y = ax + b$ and $y = cx + d$. What is $a - b - c - d$?

A. -2 B. $-\frac{1}{5}$ C. $-\frac{23}{5}$ D. $-\frac{32}{5}$ E. NOTA

14. What is the graph represented by $r = 8\cos^4\left(\frac{\theta}{4}\right) - 4\cos\left(\frac{\theta}{2}\right)$?

A. Convex Limaçon B. Limaçon with a dimple C. Cardioid D. Limaçon with an inner loop E. NOTA

15. A rose has the equation $r = \sin(\theta)\cos(\theta)\cos(2\theta)\cos(4\theta)\ldots\cos(2^n\theta)$. Let n be an integer greater than 2018. The area is given by $A\pi$ and the number of petals is given by P. Find AP in terms of n.

A. 2^{n+1} B. 2^{-1-n} C. 2^{n+2} D. 2^{-2-n} E. NOTA

16. What is the length of the latus rectum of $\frac{x^2}{4} - \frac{y^2}{9} = 1$?

A. 9 B. $\frac{9}{2}$ C. $\frac{8}{3}$ D. $\frac{4}{3}$ E. NOTA

17. A rectangle is inscribed in the region bounded by $y \geq -3x + 2, y \geq x - 3, y \leq 0$. If one edge of the rectangle lies on the x −axis, what is the area of the largest possible rectangle?

A. $\frac{97}{96}$ B. $\frac{49}{48}$ C. $\frac{25}{24}$ D. $\frac{23}{24}$ E. NOTA

18. How many of the following are true about vectors?

I. $(\vec{u} \times \vec{v}) \times \vec{w} = \vec{u} \times (\vec{v} \times \vec{w})$

II. $(c\vec{u}) \times \vec{v} = c(\vec{u} \times \vec{v}) = \vec{u} \times (c\vec{v})$

III. $\vec{w} \times (\vec{u} + \vec{v}) = \vec{w} \times \vec{u} + \vec{w} \times \vec{v}$

IV. $(\vec{u} + \vec{v}) \times \vec{w} = -\vec{w} \times \vec{u} - \vec{w} \times \vec{v}$

V. $(\vec{u} \times \vec{v}) \cdot \vec{w} = (\vec{u} \times \vec{w}) \cdot \vec{v}$

VI. $(\vec{u} \times \vec{v}) \times \vec{w} = (\vec{w} \cdot \vec{u})\vec{v} - (\vec{w} \cdot \vec{v})\vec{u}$

A. 5 B. 4 C. 3 D. 2 E. NOTA

19. Give one of the asymptotes of the hyperbola represented by the parametric equations: $x(t) = 3\sinh(t) + 2; y(t) = 4\cosh(t) + 3$.

A. $x + \frac{3}{4}y + \frac{17}{4} = 0$ B. $x - \frac{3}{4}y - \frac{17}{4} = 0$

C. $x - \frac{3}{4}y + \frac{17}{4} = 0$ D. $x + \frac{3}{4}y - \frac{17}{4} = 0$ E. NOTA

20. Find the area of the triangle formed by the points $A(0,0,0), B(1,2,3), C(e,e,1)$

A. $\frac{\sqrt{5+18e+19e^2}}{2}$ B. $\sqrt{5+18e+19e^2}$ C. $\frac{\sqrt{5-18e+19e^2}}{2}$ D. $\sqrt{5-18e+19e^2}$ E. NOTA

21. Find the area of the region bounded by $x^2 + xy + y^2 + 2x - 8y + 4 < 0$ and $x + y > 2$? (Hint: First find the center of the ellipse using the equations on the first page)

A. $16\pi\sqrt{3}$ B. $8\pi\sqrt{3}$ C. $\frac{8\pi\sqrt{3}}{3}$ D. $\frac{2\pi\sqrt{3}}{3}$ E. NOTA

22. The conic given by the polar equation $r = \frac{10}{3-2\cos(\theta)}$ is rotated by $\frac{\pi}{4}$ about the origin counterclockwise. Give the rotated conic's new equation in Cartesian form.

A. $-100 - 20\sqrt{2}x + 7x^2 - 20\sqrt{2}y - 4xy + 7y^2 = 0$

B. $-100 - 20\sqrt{2}x + 11x^2 - 20\sqrt{2}y - 4xy + 11y^2 = 0$

C. $-10 - \sqrt{2}x + 9x^2 - \sqrt{2}y - 4xy + 9y^2 = 0$

D. $10 + \sqrt{2}x + 9x^2 + \sqrt{2}y + 4xy + 9y^2 = 0$

E. NOTA

23. The orbit of Awizzle's comet, last seen in 1337 and due to return in 9001, can be modeled as an ellipse with eccentricity $\frac{9}{10}$ and one focus at the star Relpats located at the origin. Let the directrix corresponding to Relpats be vertical and to the left of Relpats on the coordinate plane. The length of its major axis is 20. Find the polar equation for the orbit of Awizzle's comet.

A. $r = \frac{1.9}{1+.9\cos(\theta)}$ B. $r = \frac{1.9}{1-.9\cos(\theta)}$ C. $r = \frac{19}{10+9\cos(\theta)}$ D. $r = \frac{19}{10-9\sin(\theta)}$ E. NOTA

24. There exists a locus of points in a plane which the sum of the distances from $(4,3)$ and some unknown other point (p, q) is 14. Furthermore, the point $(1, -1)$ is included amongst the locus of points. Determine the largest area enclosed by the different possible loci of points that fit this description.

A. $14\pi\sqrt{33}$ B. $7\pi\sqrt{33}$ C. $42\pi\sqrt{5}$ D. $21\pi\sqrt{5}$ E. NOTA

25. Which of the following is an asymptote of $3 + 11x + 10x^2 - 4y - 7xy + y^2 = 0$?

A. $y = 5x + 3$ B. $y = 2x + 3$

C. $y = 5x + 1$ D. $y = 2x - 1$ E. NOTA

26. For point $P(x_1, y_1)$ and line m with equation $ax + by + c = 0$, let point $Q(x', y')$ be the reflection of $P(x_1, y_1)$ across line m. Which of the following gives the values of (x', y') in terms of a, b, c, x_1, y_1?

A. $\left(x_1 + \frac{2a(ax_1+by_1+c)}{a^2+b^2}, y_1 + \frac{2b(ax_1+by_1+c)}{a^2+b^2}\right)$ B. $\left(x_1 + \frac{a(ax_1+by_1+c)}{a^2+b^2}, y_1 + \frac{b(ax_1+by_1+c)}{a^2+b^2}\right)$

C. $\left(x_1 - \frac{2a(ax_1+by_1+c)}{a^2+b^2}, y_1 - \frac{2b(ax_1+by_1+c)}{a^2+b^2}\right)$ D. $\left(x_1 - \frac{a(ax_1+by_1+c)}{a^2+b^2}, y_1 - \frac{b(ax_1+by_1+c)}{a^2+b^2}\right)$

E. NOTA

27. Let C be the graph of $-2x + x^2 - xy + y + 2y^2 = 4$, and denote by C' the reflection of C in the line $y = -x + 1$. Let the equation of C' be written in the form $x^2 + bxy + cy^2 + dx + ey + f = 0$, Find $b + c + d + e + f$.

A. -5 B. 5 C. -4 D. -3 E. NOTA

28. Two mutually perpendicular chords are drawn from the vertex of a parabola such that their lengths are 12 and 30. Which of the following is closest to the length of the latus rectum? (Hint: This is possible for only one distance between the parabola's focus and the directrix)

A. $\frac{36}{5}$ B. $\frac{144}{11}$ C. $\frac{20736}{25}$ D. $\frac{6}{\sqrt{5}}$ E. NOTA

29. What is the area in the Argand plane enclosed by the curve $|z| < 2018$?

A. $1009^2\pi$ B. 2018 C. 2018^2 D. 2018π E. NOTA

30. What is the distance from the origin to $x^2 + 2x + 2018y^2 = -1$?

A. 1 B. $\sqrt{2}$ C. $\sqrt{2018}$ D. 2018 E. NOTA